UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCES

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE: **MATHEMATICS FOR DIPLOMA STUDENTS – MFD001**

CAMPUS: **SWC**

ASSESSMENT: **EXAM**

DATE: **29 OCTOBER 2015**

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DURATION: **2 HOURS**

70

INITIALS AND SURNAME: _____

STUDENT NUMBER: _____

CONTACT NUMBER: _____

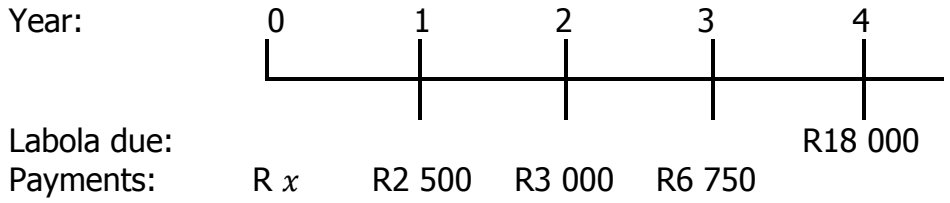
NUMBER OF PAGES: 15 (INCLUDING COVER PAGE AND ANNEXURE A)

INSTRUCTIONS:

- **ANSWER ALL THE QUESTIONS IN PEN**
- **NO REMARK ON PENCIL, NO TIPEX ALLOWED**
- **ALL GRAPHS MUST BE DRAWN IN PEN**
- **STATE ALL FORMULAS USED, MARKS ARE GIVEN TO FORMULAS**
- **SHOW ALL THE NECESSARY CALCULATIONS**
- **IF NECESSARY ROUND OFF TO TWO DECIMAL PLACES**
- **SCIENTIFIC CALCULATORS ARE ALLOWED**
- **QUESTIONS CAN BE ANSWERED IN ANY ORDER**

Question 1 **[6]**

Titus wants to marry Mary but her traditional father insists that he must first pay R18 000 labola. Titus negotiates with the father to pay the labola over a few years as he is still searching for a decent job. They agree to the following payments:



If money is worth 8% compounded quarterly, determine how much the first payment of x must be.

Question 2 **[12]**

A borrower is repaying a R750 000 loan at 6.5% per year, compounded monthly, with monthly payments over 25 years.

2.1 Determine the monthly payment. (3)

2.2 Determine the balance outstanding after the 100th payment. (3)

2.3 Determine the interest contained in the 10th payment. (3)

2.4 Determine the finance charge. (3)

Question 3 **[2]**

Graph the following inequality and clearly indicate the feasible region:

$$3y > 8x - 24$$

Question 4 **[3]**

For the following word problem, only write down the system of constraints.

A producer grower is purchasing fertilizer containing three ingredients, A, B, and C. The minimum needs are 160 units of A, 200 units of B, and 80 units of C. There are two popular brands of fertilizer on the market. Fast Grow, costing $8 a bag, contains 3 units of A, 5 units of B, and 1 unit of C. Easy Grow, costing $6 a bag, contains 2 units of each nutrient.

Question 5 **[6]**

Let x be the number of litres of regular petrol and y the number of litres of premium petrol. Given the following constraints:

$$3x + 5y \leq 120$$
$$6x + 4y \leq 180$$
$$x \geq 0, y \geq 0$$

As well as the objective function: $P = 80x + 120y$

5.1 Graph the feasible region. (4)

5.2 Determine the value(s) of x and y such that P is a maximum. Also give the maximum value of P. (2)

Question 6 **[3]**

For the following equation:

$$2x^3 + 7y = 16$$

Determine $\frac{dx}{dy}$.

Question 7 **[10]**

Differentiate each of the following functions with respect to x. You do not need to simplify your solution.

7.1 $f(x) = x^3 + 7x^{-2} + 18 - \sqrt{x}$ (2)

7.2 $g(x) = \ln(2x^3 + 2x - 6)$ (2)

7.3 $h(x) = \frac{4x^2+3x-9}{e^{2x}-1}$ (3)

7.4 $j(x) = (2x^3 + 2x - 6)^{20} . (1 - x^2)^{-3}$ (3)

Question 8 **[7]**

Given the following data set:

7 10 9 6 4 7 6

8.1 Determine the mean, mode and median. (3)

8.2 Determine the standard deviation after completing the table below. (4)

i	x_i	$x_i - mean$	$(x_i - mean)^2$
1.	7		
2.	10		
3.	9		
4.	6		
5.	4		
6.	7		
7.	6		
		TOTAL	

Question 9 [2]

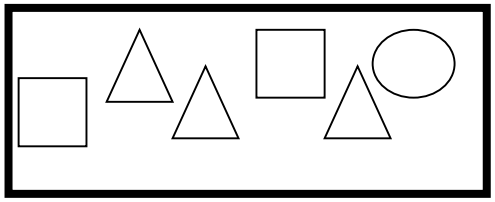
Let S be the sample space of a die, and define the following three events on S:

$A = \{\text{even numbers}\}$ $B = \{\text{odd numbers}\}$ $C = \{3,6\}$

Draw a Venn diagram to illustrate S, A, B and C.

Question 10 [6]

The following box contains 3 triangles, 2 squares and 1 circle:



You draw two shapes from the box at random; first the one and then the second. If replacement is **NOT** allowed,

10.1 Draw a complete tree diagram illustrating all possibilities and probabilities. (3)

10.2 Determine the probability of first drawing a circle and then a triangle. (1)

10.3 Determine the probability of drawing two identical shapes. (2)

Question 11 [6]

A few students at UJ filled out an entry form for a competition to win an iPod. Here are their names and their ages:

Boys	Ages	Girls	Ages
Nathan	18	Nomsa	18
Marie	19	Francine	18
Jonathan	22	Delia	23
Mutumi	24	Gracie	27
John	17	Petro	19
David	25	Violet	18
Rafael	25	Morwesi	25
		Charisa	24

Each student has the same probability of winning, and there can only be one winner.

Given the following events:

E_1 : *A boy wins*
E_2 : *An 18-year-old wins*
E_3 : *The winner's name starts with an "N"*
E_4 : *A 25-year-old wins*

Determine:

11.1 $P(E_4)$ (1)
11.2 $P(\overline{E_4})$ (1)
11.3 $P(E_2 \cap E_4)$ (1)
11.4 $P(E_2 \cap E_3)$ (1)
11.5 $P(E_1 \cup E_4)$ (2)

Question 12 (Round off to 4 decimal places here) [7]

One of your investments on the Johannesburg Stock Exchange (JSE) is currently trading around a mean of \$10 with a standard deviation of \$10.

12.1 Calculate the probability that your stock will trade below \$5. (3)

12.2 Calculate the probability that your stock will trade between \$13 and \$25.50. (4)

End of Assessment – Total Marks: 70

ANNEXURE A

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990